

Private Release of Graph Statistics using Ladder Functions

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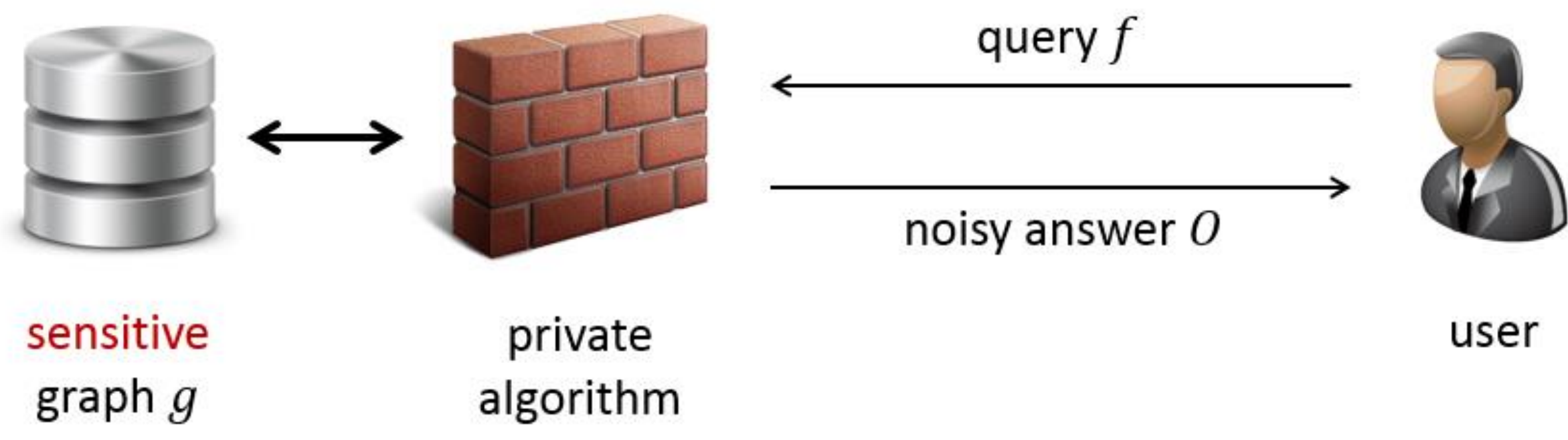
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1. Private Release of Graph Statistics

Private Data Release



Privacy: O should reveal little about any relationship in g

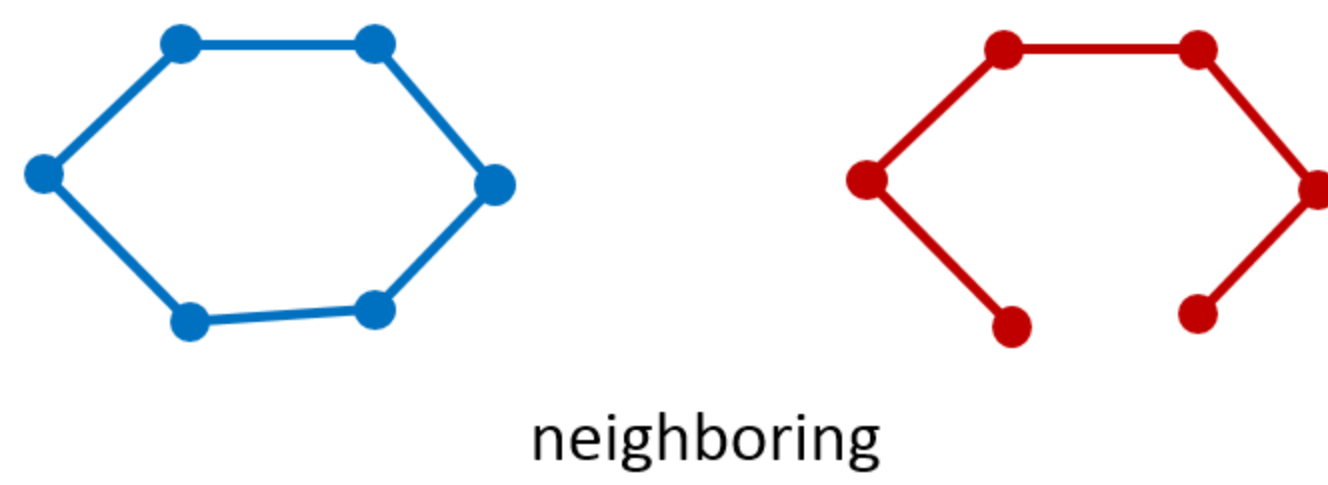


Utility: O should be as accurate as possible

Differential Privacy on Graph

Differentially private algorithm injects noise into the query answer, in order to cover the **maximum impact of a relationship (an edge)**.

query1: are nodes a and b connected?



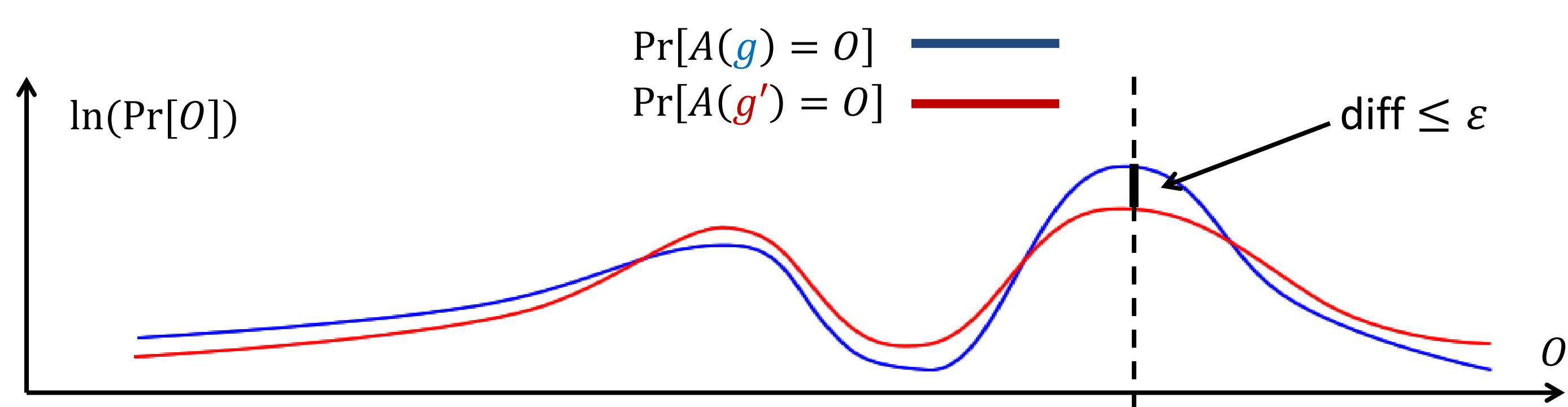
query2: how many edges?

query3: how many triangles?



2. Global and Local Sensitivity

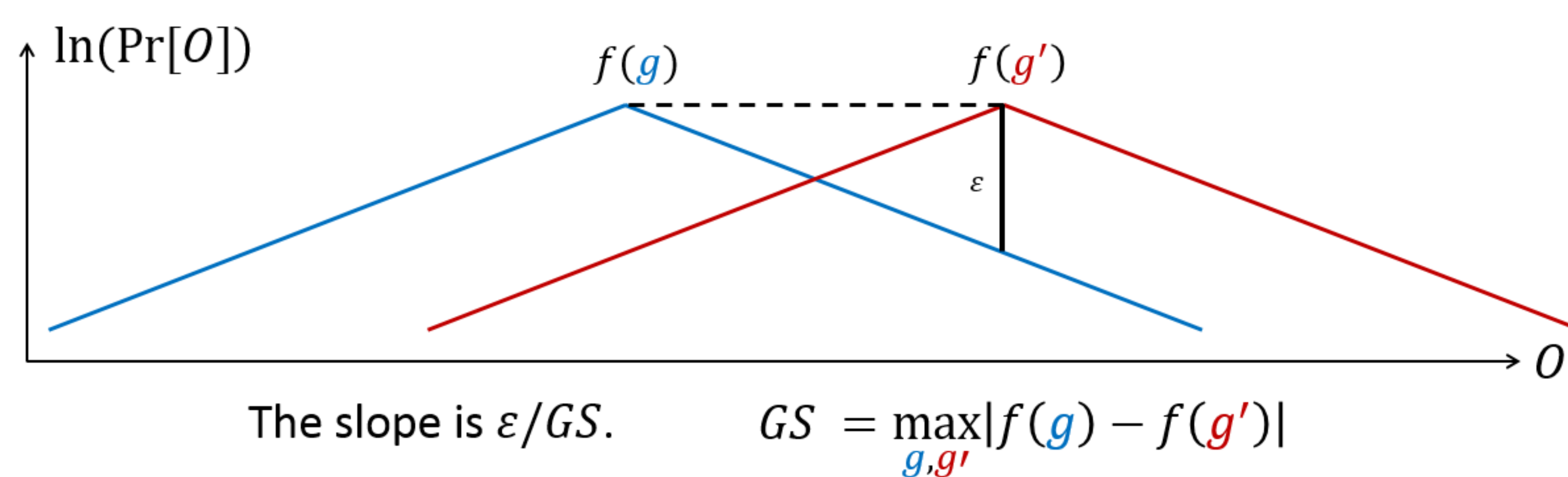
Formal Definition of Differential Privacy



where g and g' are neighboring graphs that differ by **at most one edge**

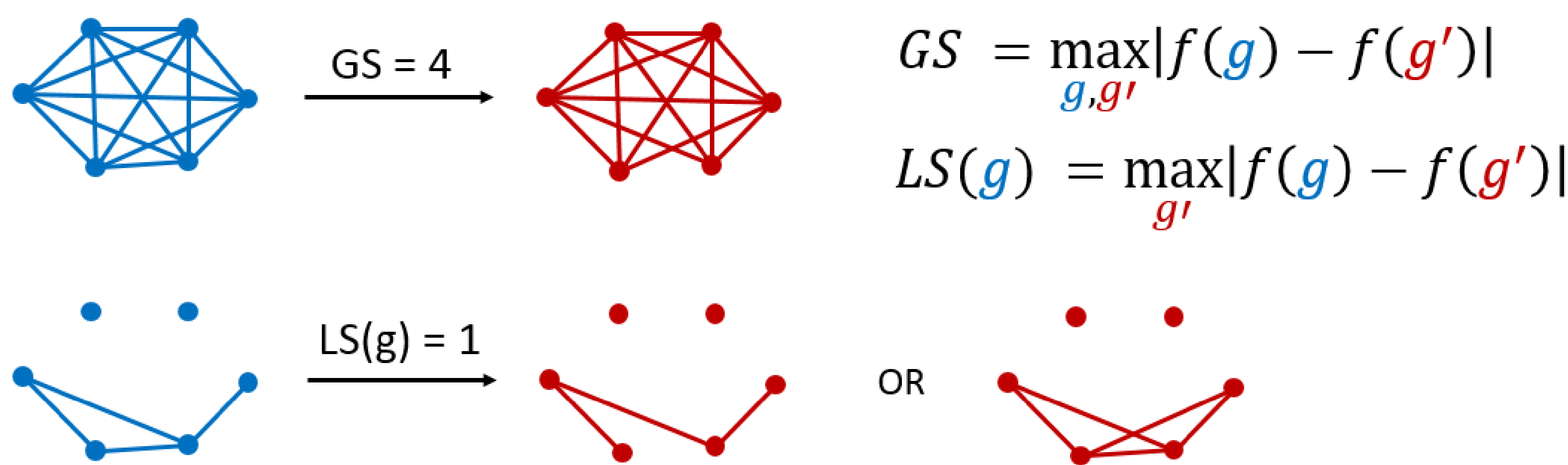
$$|\ln(\Pr[A(g) = O]) - \ln(\Pr[A(g') = O])| \leq \epsilon$$

Global Sensitivity



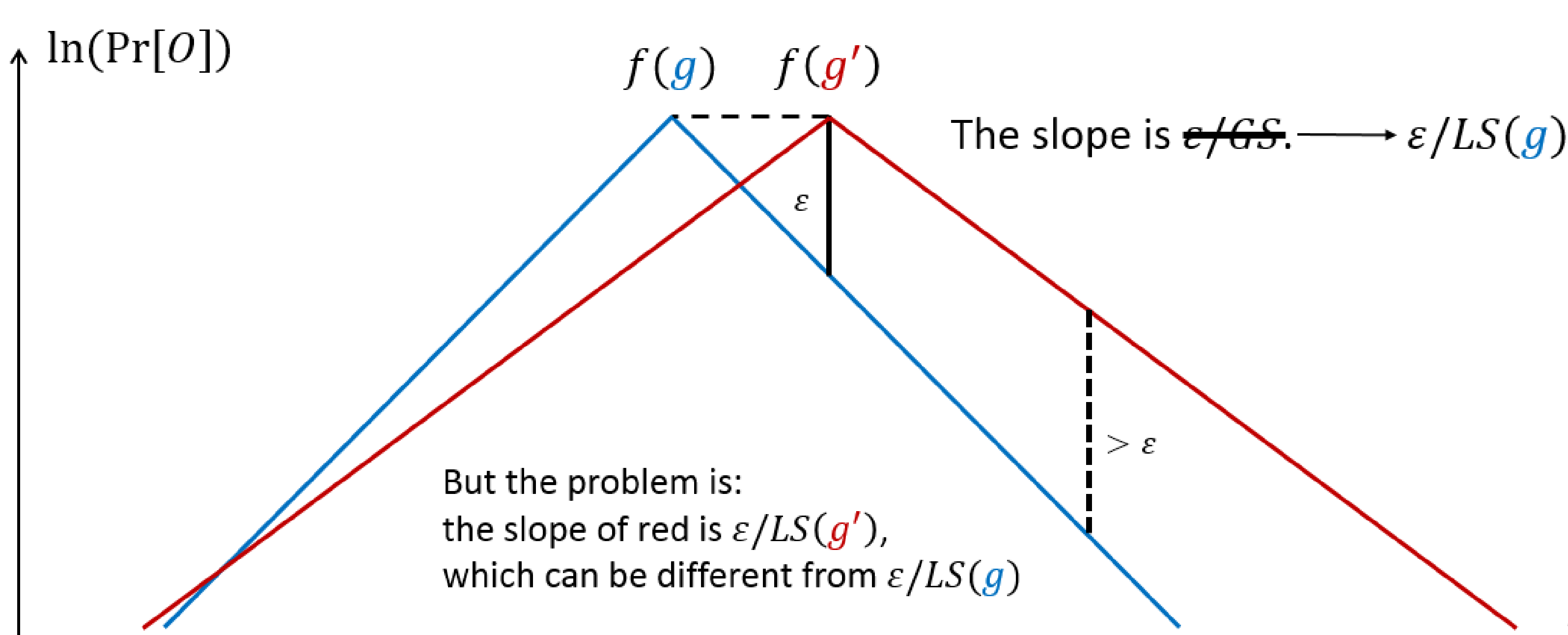
The slope is ϵ/GS . $GS = \max_{g, g'} |f(g) - f(g')|$

Local Sensitivity



$$GS = \max_{g, g'} |f(g) - f(g')|$$

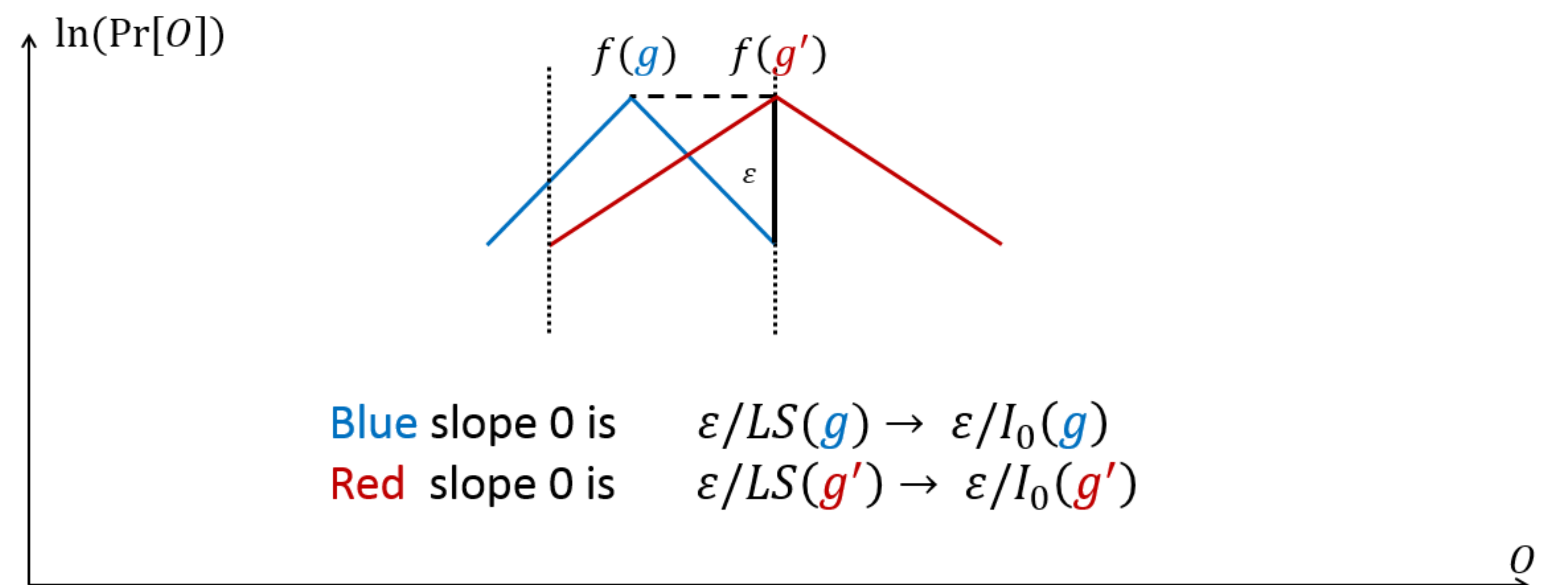
$$LS(g) = \max_{g'} |f(g) - f(g')|$$



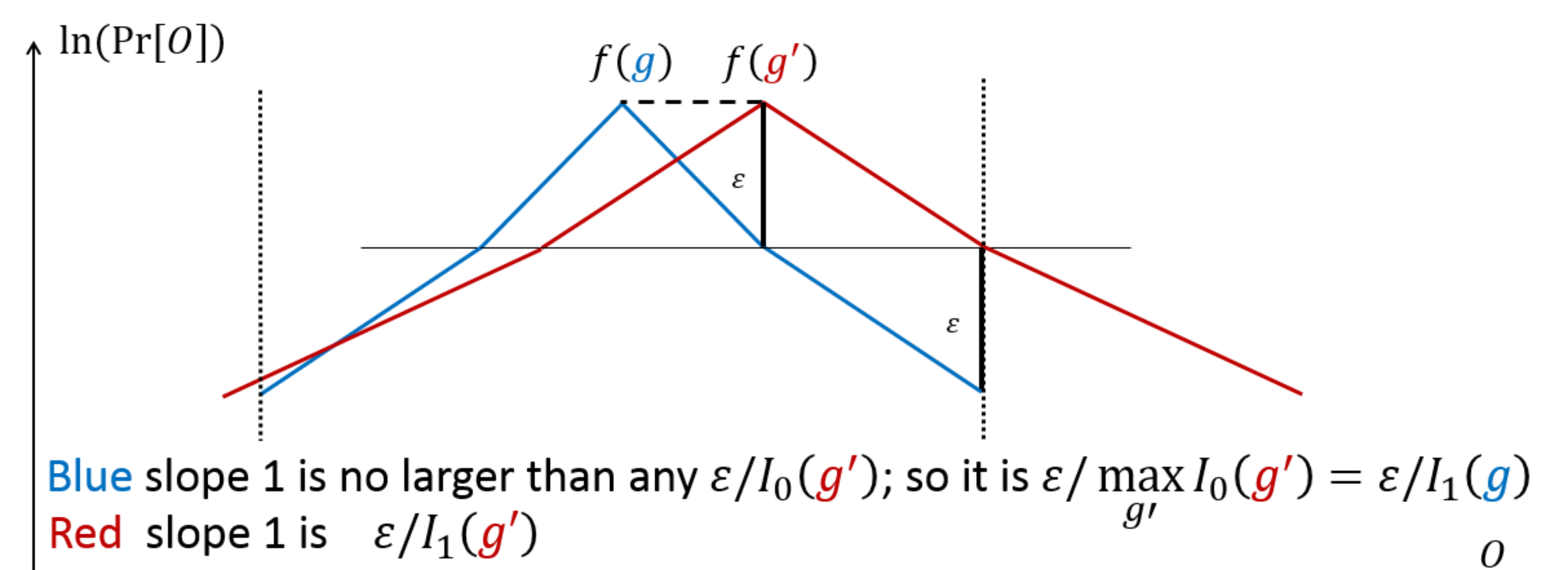
But the problem is: the slope of red is $\epsilon/LS(g')$, which can be different from $\epsilon/LS(g)$

3. Ladder Functions

Key Idea: Change Slope Gradually

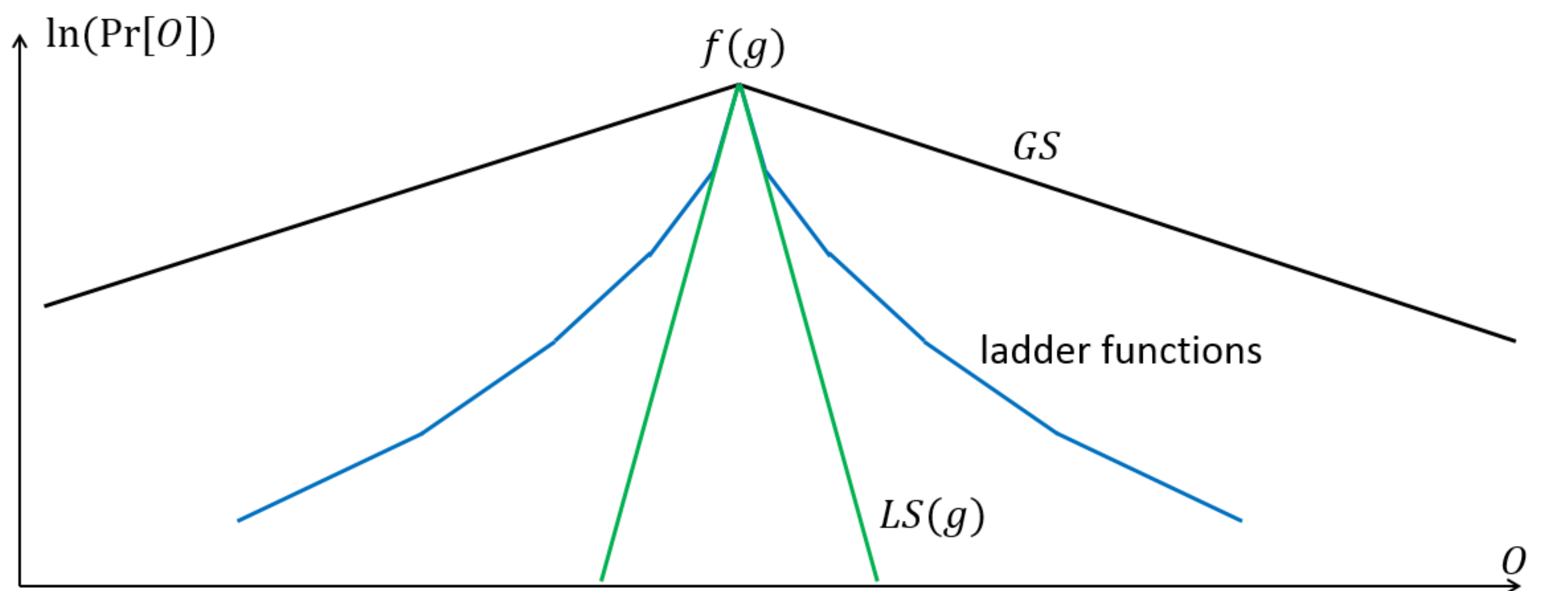


Blue slope 0 is $\epsilon/LS(g) \rightarrow \epsilon/I_0(g)$
Red slope 0 is $\epsilon/LS(g') \rightarrow \epsilon/I_0(g')$

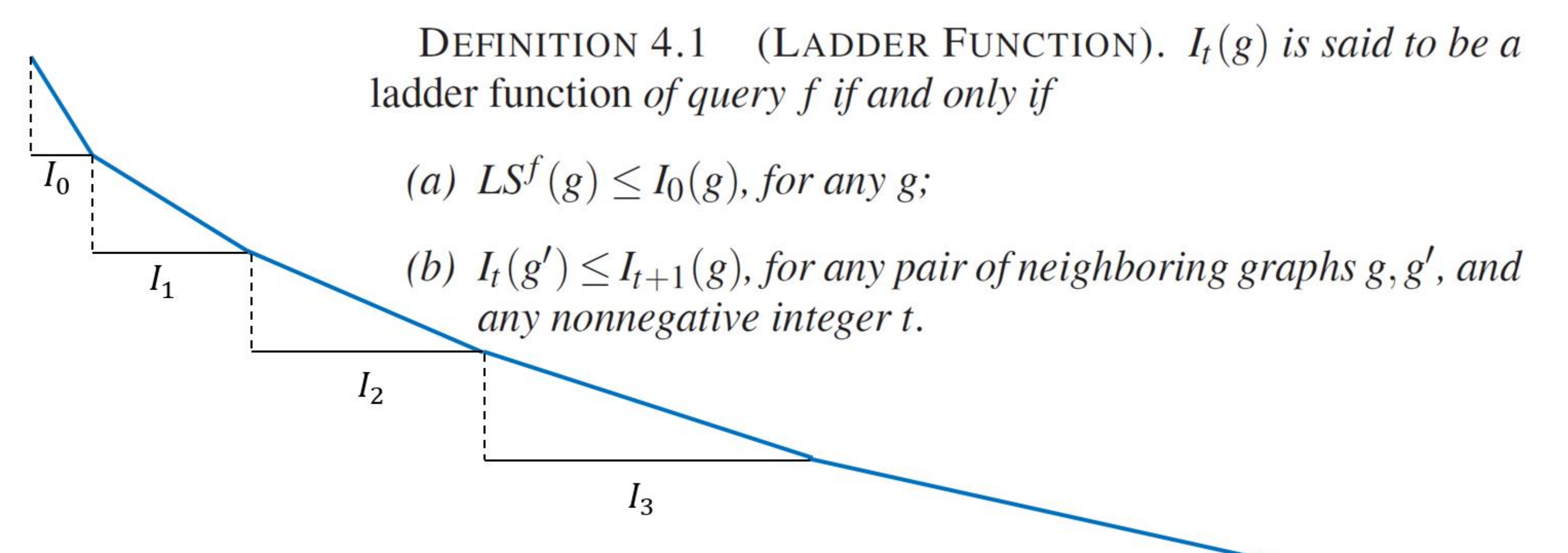


Blue slope 1 is no larger than any $\epsilon/I_0(g')$; so it is $\epsilon/\max_{g'} I_0(g') = \epsilon/I_1(g)$
Red slope 1 is $\epsilon/I_1(g')$

Summary



Formal Results



DEFINITION 4.1 (LADDER FUNCTION). $I_t(g)$ is said to be a ladder function of query f if and only if

- (a) $LS^f(g) \leq I_0(g)$, for any g ;
- (b) $I_t(g') \leq I_{t+1}(g)$, for any pair of neighboring graphs g, g' , and any nonnegative integer t .

4. Experiments

